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# Thermal conductivity of Newtonian and non-Newtonian liquids

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ABSTRACT

In the present paper the results of the experimental studies directed to the correlation between the non-Newtonian fluids rheological behaviour and thermal conductivity taking into account shear rate effect, have been presented. In the experimental tests 3 Newtonian liquids as well as 18 aqueous solutions of polymers, have been used. Experiments were performed at the solutions concentrations ranged from 1000 to 5000 w.ppm and at temperature changed from 299 to 315 K. The shear rate range extended $0 \leq \dot{\gamma} \leq 750$ ($s^{-1}$). For the non-Newtonian aqueous solutions studied the relation of $\lambda = f(T, \dot{\gamma})$ is evident and should be taken into account in all design practices.

## 1. Introduction

Many important industrial fluids, which are non-Newtonian or rheologically complex in their flow characteristics are often used in the chemical processes and food industries, as well as in many other practical applications. For the unavailability of another literature data in previous works on non-Newtonian fluid heat transfer the thermal conductivity was accepted to be identical with that one in the clear solvent. It was essentially a rough approximation that existed long. Additionally, it must be emphasized that until now the kinetic-molecular theory for liquids does not exist, and the relationships proposed for thermal conductivity in Newtonian liquids are empirical and very different in their form (Table 1).

Of the significance is the fact that in non-Newtonian fluids the most experimental studies on thermal conductivities were performed under static conditions [6,7]. Bellet et al. [6] have determined the volume specific heat at constant pressure and thermal conductivity using an analytical identification method (Fig. 1). Authors showed that the volume specific heat is independent of shear rate. In both, carboxymethylcellulose sodium salt solution (Na-CMC RC 197) and Carbopol 960 aqueous solutions in the range of temperature $293 \leqslant T \leqslant 333$ K thermal conductivity increased with increasing of temperature and decreased with an increase of polymer concentration in a solution (Fig. 2).

The study was summarized by the following relations:

- for Na-CMC RC 197 aqueous solutions

$$\frac{K}{\eta} = 15.59\Delta\lambda^{0.1534} \Rightarrow \Delta\lambda = 1.670 \times 10^{-8}\left(\frac{K}{\eta}\right)^{6.52} \tag{6}$$

- for Carbopol 960 aqueous solutions

$$\frac{K}{\eta} = 13.46\Delta\lambda^{0.2306} \Rightarrow \Delta\lambda = 1.259 \times 10^{-5}\left(\frac{K}{\eta}\right)^{4.34} \tag{7}$$

- for both polymer solutions tested

$$\lambda_{C_p=const} = AT + B \tag{8}$$

where

$$\Delta\lambda = \lambda_s - \lambda \tag{9}$$

$K$ represents the consistency factor of a solution, $\eta$ is the viscosity of a solvent, $\lambda_s$ and $\lambda$ are the thermal conductivities of solvent and polymer solution, respectively. The correlation values of $A$ and $B$ are presented in Table 2.

Lee et al. [7] have tested the aqueous solutions of polyethylene oxide, polyacrylamide, carboxymethylcellulose sodium salt, Carbopol 960 and Attagel 40 (specialty attapulgite product, fine-milled grade, thickener of waterborne systems) in the range of concentrations changed from 100 to 10,000 w.ppm under static conditions. As a result they stated an increase of thermal conductivity when the concentration of the polymer in a solution increased, thus indicating the contrary effect in reference to the study presented above [6].

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## Nomenclature

| | |
|---|---|
| $A,B$ | constants |
| $C_p$ | concentration of polymers in a solution (w.ppm) |
| $c$ | heat capacity (J kg$^{-1}$ K$^{-1}$) |
| $d_i$ | diameter of inner cylinder (m) |
| $d_o$ | diameter of outer cylinder (m) |
| $I$ | current intensity (A) |
| $K$ | consistency factor (Pa s$^n$) |
| $k_B$ | Boltzmann constant (J K$^{-1}$) |
| $L$ | length of the main heating region (m) |
| $M$ | molar mass of a polymer (kg kmol$^{-1}$) |
| $n$ | flow behaviour index |
| $Q$ | heat flux (W) |
| $T$ | mean temperature between inside and outside cylinders (K) |
| $u_p$ | mass fraction of polymers in a solution (kg p kg$^{-1}$) |
| $V$ | voltage (V) |
| $V_D$ | acoustic velocity in the liquid (m s$^{-1}$) |
| $Z$ | mean distance between the centres of the molecules (m) |

*Greek symbols*

| | |
|---|---|
| $\dot{\gamma}$ | shear rate (s$^{-1}$) |
| $\eta$ | viscosity of a solvent (Pa s) |
| $\rho$ | density (kg m$^{-3}$) |
| $\rho_w$ | specific density relative to water |
| $\lambda$ | thermal conductivity (W m$^{-1}$ K$^{-1}$) |
| $\lambda_0$ | thermal conductivity in static conditions (W m$^{-1}$ K$^{-1}$) |
| $\tau$ | shear stress (Pa) |

The measurements of non-Newtonian fluid thermal conductivity should be made when the fluid is in a motion. Only few previous works studying the effect of shear rate on thermal conductivity in non-Newtonian polymer solutions have been found, whereas their results are contradictory (Cocci and Picot [8]; Chitranged and Picot [9]; Picot et al. [10]; Wallace et al. [11]; Loulou et al. [12]; Chaliche et al. [13]; Shin [14]; Lee and Irvine [15]; Kostic and Tong [16]; Shin and Lee [17] as well as Lin et al. [18]). The exemplary graphs of the shear rate dependent thermal conductivity of non-Newtonian fluids resulted from these studies are presented in Fig. 3 (Lee and Irvine [15]).

Cocci and Picot [8] showed that the thermal conductivity in Dow 200 fluids increased increasing shear rate over the region of $0 \le \dot{\gamma} \le 400$ s$^{-1}$. Authors also found that the rate of thermal conductivity increase for Dow 200 fluids decreased when increasing the temperature. They reported that thermal conductivity in polymer materials increases with molecular weight of a polymer. Chitrangad and Picot [9] and Picot et al. [10] have stated that increasing the shear rate in a low region ($\dot{\gamma} \le 200$ s$^{-1}$) thermal conductivity of Dow 200 fluids and polyethylene melts increased up to a maximum point and then decreased in the region $200 \le \dot{\gamma} \le 400$ s$^{-1}$.

Picot et al. [10] have also reported the increase in thermal conductivities of polymeric materials with raise of their molecular weight. Thermal conductivity was measured at temperatures approaching 200 °C and shear rates up to 400 s$^{-1}$. Polyethylene melts were studied. The results showed that at 50 s$^{-1}$, an 2% increase in thermal conductivity occurs followed by a gradual decrease of 10% from its no-shear value at shear rate of 400 s$^{-1}$. This effect was interpreted by peculiarities in the molecular orientation. On the other hand, Wallace et al. [11] have observed that, depending on polymer's molecular weight there existed either an increase, or a decrease in values of thermal conductivity of polyethylene melts with shear rate in the region $0 \le \dot{\gamma} \le 400$ s$^{-1}$. The polymer melt of smaller molecular weight showed the 55% decrease with shear rate, while the melts of greater molecular weight first indicated a decrease until reaching the minimum, later on an increase in thermal conductivity at higher shear rates.

Loulou et al. [12] and Chaliche et al. [13] have noted slight changes in shear rate dependent thermal conductivity attaining 3% at the shear rate of 20 s$^{-1}$ for Carbopol solutions (1000 and 2000 w.ppm) and 5% at the shear rate of 50 s$^{-1}$ for carboxymethylcellulose solutions sodium salt (30,000–80,000 w.ppm). Additionally they observed that at the lower concentrations of the polymer in a solution, the larger changes in thermal conductivity took place. Chaliche et al. [13] showed a gradual increase of thermal conductivity with shear rate depending either on temperature or on polymer concentration. A higher temperature as well a lower concentration in a solution bring about greater thermal conductivities.

**Table 1**
Basic relationships for determination of the thermal conductivity for Newtonian liquids.

| Author | Relationship | |
|---|---|---|
| Weber [1] | $\lambda = 3.58 \cdot 10^{-8} c\rho \sqrt[3]{\frac{\rho}{M}}$ | (1) |
| Bridgeman [2] | $\lambda = \frac{3k_B V_D}{Z^2}$ | (2) |
| Kardos [3] | $\lambda = 9.510^{-11} c\rho V_D$ | (3) |
| Smith [4,5] | $\lambda = \frac{0.362 \cdot 10^{-6} \rho_w^{2.15} c^{1.55} M^{0.192}}{\eta^{0.12}}$ | (4) |
| | $\lambda = 1.733\left[0.00266 + 1.56\left(\frac{c}{4186.8} - 0.45\right)^3 + 0.3\left(\frac{\rho_w}{M}\right)^{0.33} + 0.0521\left(\frac{\eta}{\rho_w}\right)^{0.11}\right]$ | (5) |

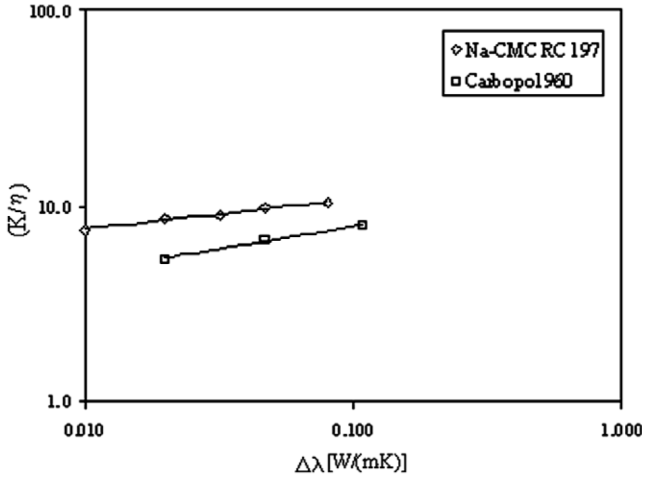

**Fig. 1.** Change of thermal conductivity in non-Newtonian polymer aqueous solutions [6].

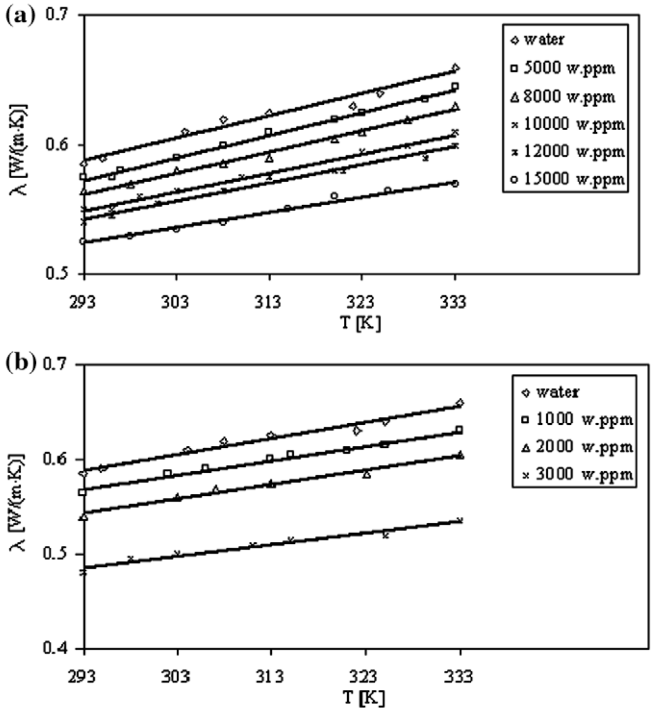

**Fig. 2.** Effect of solution concentration and temperature on thermal conductivity [6]: (a) carboxymethylcellulose sodium salt Na-CMC RC 197 and (b) Carbopol 960.

**Table 2**
Values of coefficients *A* and *B* in Eq. (8) proposed by Bellet et al. [6].

| | $C_p$ [w.ppm] | *A* | *B* |
|---|---|---|---|
| Water | 0 | 0.0017 | 0.0876 |
| Na-CMC RC 197 | 5000 | 0.0018 | 0.0598 |
| | 8000 | 0.0016 | 0.0872 |
| | 10,000 | 0.0015 | 0.1210 |
| | 12,000 | 0.0014 | 0.1326 |
| | 15,000 | 0.0012 | 0.1717 |
| Carbopol 960 | 1000 | 0.0015 | 0.1205 |
| | 2000 | 0.0015 | 0.0984 |
| | 3000 | 0.0012 | 0.1286 |

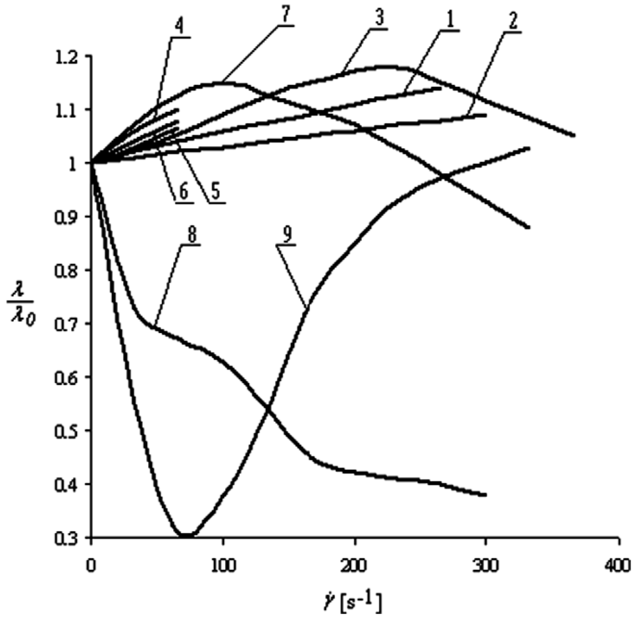

**Fig. 3.** Comparison of the shear rate dependent thermal conductivity of non-Newtonian fluids resulted from literature (Lee and Irvine [15]): 1 – Cocci and Picot [8], Dow 200, *T* = 288.7 K; 2 – Cocci and Picot [8], Dow 200, *T* = 327 K; 3 – Chitrangad and Picot [9], Dow 200, *T* = 298 K; 4 – Chaliche et al. [13], Na-CMC, 8%, *T* = 298 K; 5 – Loulou et al. [12], Carbopol, 0.1%, *T* = 298 K; 6 – Loulou et al. [12], Carbopol, 0.2%, *T* = 298 K; 7 – Picot et al. [10], polyethylene melt, *T* = 423 K; 8 – Wallace et al. [11], low molecular weight polyethylene melt, *T* = 433 K; 9 – Wallace et al. [11], high-molecular weight polyethylene melt, *T* = 433 K.

Lee and Irvine [15] researched the aqueous solutions of Na-CMC at the concentrations of 1500, 2500 and 5000 w.ppm as well as of Separan AP-273 (polydisperse, high-molecular weight, $10^5$–$10^7$ Dalton range, anionic polyacrylamide) at the concentrations of 1000 and 2000 w.ppm. They observed that by increasing the shear rate for both, Na-CMC ($100 \le \dot{\gamma} \le 900\ \mathrm{s}^{-1}$) and Separan ($50 \le \dot{\gamma} \le 600\ \mathrm{s}^{-1}$), solutions the thermal conductivity increased by an amount of 20–70% and 20–50%, respectively, depending on temperature (293–323 K). The increase in thermal conductivity was greater at lower concentration of solutions. The thermal conductivity of both polymer solutions increased with the increase of temperature and shear rate.

In the study of Kostic and Tong [16] the 1000 and 2000 w.ppm polyacrylamide (Praestol 2273) solutions were examined at various shear rates ($50 \le \dot{\gamma} \le 510\ \mathrm{s}^{-1}$) and at temperature 300 K. The increase in thermal conductivity with shear rate was more

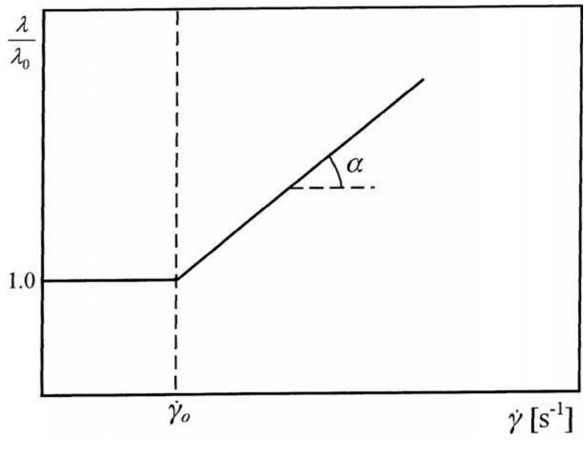

**Fig. 4.** Theoretical model of thermal conductivity for non-Newtonian fluids [14].

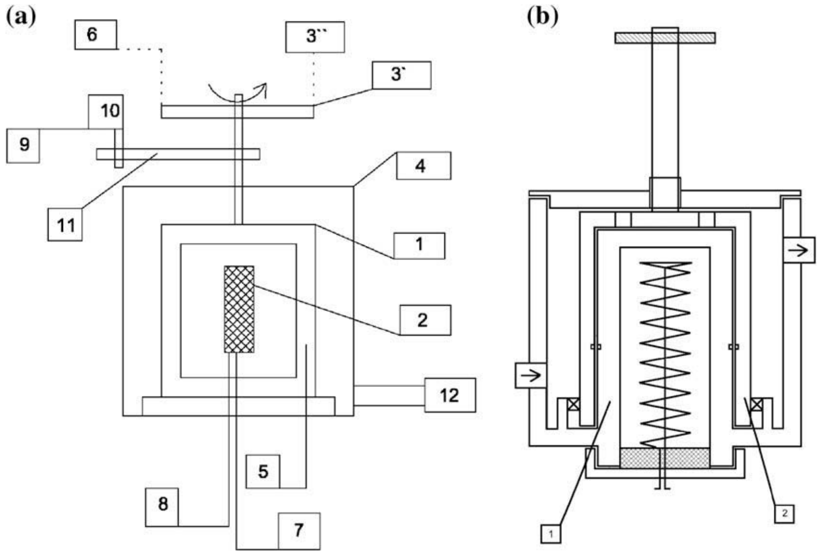

**Fig. 5.** Scheme of the experimental set-up (a) and coaxial cylinders used (b): 1 – coaxial cylinders, 2 – heater, 3 – integrated circuit to measure temperature of outer cylinder, 4 – water jacket, 5 – meter to measure temperature of inner cylinder, 6 – tachometer, 7 – power supply, 8 – ammeter/voltmeter, 9 – motor speed controller, 10 – motor, 11 – rotating mechanism, 12 – water bath.

noticeable in the lower concentrated (1000 w.ppm, 17%) Praestol solution in comparison with the higher concentrated one (2000 w.ppm, 13%). Kostic and Tong [16] have suggested that due to complexities of the phenomena and difficulties involved in the study, further improvements of the apparatus, instrumentation and procedure, as well as much more measurements for different fluids and temperature levels, and over wider range of shear rates, are needed.

**Table 3**
Newtonian liquids used.

| Name | $\rho$ (kg m$^{-3}$) | $\eta \times 10^3$ (Pa s) | Authors' experimental data correlations |
|---|---|---|---|
| Diesel oil | 844.0–849.2 | 2.76–3.53 | $\lambda = 33.5\ T^{-0.803}$ |
| Water | 991.5–996.8 | 0.630–0.900 | $\lambda = 1.11 \times 10^{-5}\ T^{1.91}$ |
| Aqueous solution of glycerol (50%) | 1116–1125 | 2.06–5.30 | $\lambda = 1.57 \times 10^{-7}\ T^{2.6}$ |

Shin and Lee [17] have experimentally investigated the rheological behaviour and the thermal conductivity of suspensions of polyethylene and polypropylene by examining the effects of shear rate, particle size, and volume concentrations. Four different sizes of plastic particles (25–300 μm) were used in solid-liquid suspensions. The volume concentration of the particles and shear rate varied within the ranges of 0–10% and 0–900 s$^{-1}$, respectively. Authors found that for the suspensions of large particles ($d \geqslant 100$ μm), the thermal conductivity increased with shear rate, implying its strong dependence on the size of dispersed particles. A shear rate dependence of thermal conductivity is increased with volume concentration of solid particles. Shin and Lee [17] proposed a new correlation for shear rate dependent thermal conductivity of suspensions including the effects of volume concentration and particle size. Lin et al. [18] described thermal conductivity measurements of two concentrated fruit juices (orange and mango) at various shear rates ($0 \leq \dot{\gamma} \leq 1100\ \mathrm{s}^{-1}$) and three temperatures: 305, 315 and 325 K. It has been found that the thermal conductivity for both fluids increases as the shear rate increases.

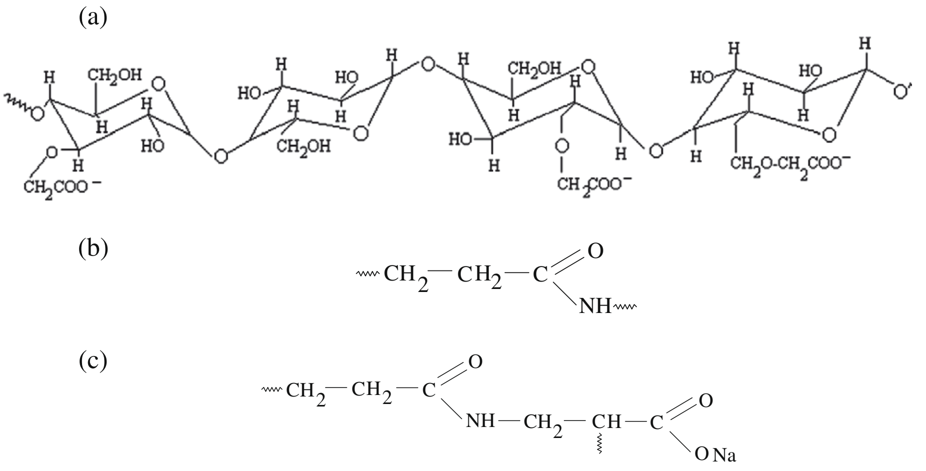

**Fig. 6.** Polymers dissolved in distilled water used in the study: (a) carboxymethylcellulose sodium salt, (b) polyacrylamide, Rokrysol WF1, and (c) polyacrylamide, Rokrysol WF2.

**Table 4**
Flow behaviour index and consistency coefficient values for the non-Newtonian non-elastic power-law aqueous polymer solutions tested.

| Polymer | $M_p$ (kg kmol$^{-1}$) | $u_p$ (kg p kg$^{-1}$) | $n$ | $K$ (Pa s$^n$) |
|---|---|---|---|---|
| Na-CMC | 250,000 | 0.001 | 0.90–0.98 | 0.0030–0.0068 |
| | | 0.003 | 0.87–0.95 | 0.0062–0.0147 |
| | | 0.005 | 0.85–0.95 | 0.0100–0.0279 |
| Na-CMC | 700,000 | 0.001 | 0.53–0.63 | 0.1041–0.2736 |
| | | 0.003 | 0.56–0.61 | 0.2383–0,4311 |
| | | 0.005 | 0.51–0.58 | 0.5628–1.1397 |
| PAA, Rokrysol WF1 | (2–4) $10^6$ | 0.001 | 0.66–0.71 | 0.0124–0.0139 |
| | | 0.003 | 0.61–0.63 | 0.0290–0.0358 |
| | | 0.005 | 0.60–0.66 | 0.0308–0.0552 |
| PAA, Rokrysol WF2 | (2–4) $10^6$ | 0.001 | 0.63–0.73 | 0.0100–0.0246 |
| | | 0.003 | 0.53–0.63 | 0.0328–0.0769 |
| | | 0.005 | 0.52–0.59 | 0.0535–0.1044 |

**Table 5**
Aqueous solutions of the polymers mixed used in the studies.

| Mixture | Mass fraction of the polymers in a mixture (kg p kg$^{-1}$) | | | $n$ | $K$ (Pa s$^n$) |
|---|---|---|---|---|---|
| | Na-CMC, $M$ = 700,000 (kg kmol$^{-1}$) | Rokrysol WF1 | Rokrysol WF2 | | |
| I | 0.001 | 0.001 | – | 0.55–0.65 | 0.0940–0.2449 |
| II | | – | 0.001 | 0.56–0.63 | 0.1226–0.2643 |
| III | | 0.005 | – | 0.57–0.63 | 0.0867–0.1951 |
| IV | 0.005 | 0.001 | – | 0.54–0.60 | 0.4607–0.8892 |
| V | | – | 0.005 | 0.52–0.58 | 0.5577–1.0632 |
| VI | | 0.005 | – | 0.53–0.59 | 0.5356–1.0434 |

Shin [14] has presented theoretical study where the empirical linear model for shear rate dependent thermal conductivity was adopted (Fig. 4). The model can be formulated as follows:

$$\lambda = \lambda_0 + (\dot{\gamma} - \dot{\gamma}_0) tg\alpha \tag{10}$$

where $\lambda_0$ represents the stationary thermal conductivity of fluid, and $\dot{\gamma}_0$ indicates the bound of the linear region below of which the thermal conductivity seems to be invariable and independent of shear rate.

The present study is concerned with determination of viscous properties of chosen non-Newtonian liquids with a view to correlate the viscous and thermal properties of these fluids. In the fluids studied the thermal conductivity was measured both, in static and dynamic conditions, over a shear rate range where the viscous properties significantly vary with shear rate.

## 2. Experimental set-up

To measure the heat conductivity the new experimental installation was designed (Fig. 5) which permitted to determine thermal conductivity coefficient for power-law fluids basing on the rheometric technique and very accurate temperature measurement on outer cylinder.

The main parts of the system were two coaxial cylinders while the inner cylinder is mounted as stationary and the outer cylinder is rotating. Cylinders made of steel chosen as the one permitted to use in food technology are of the following dimensions: outer cylinder with inner diameter 72 mm and inner cylinder with outer diameter 69 mm. The test fluid was located in the annular gap of 1.5 mm between two cylinders. The inner cylinder included the resistance sensors and the heater (main and two guards). Three calibrated sensors were used to measure the surface temperature of the inner cylinder. The resistance sensors were positioned in the middle of the test section, and uniformly distributed at 120° interval. Analogically, three sensors were mounted in the surface of the outer cylinder.

The novel element of experimental set-up to measure the temperature of the outer cylinder consists of two parts as integrated circuits: transmitter and receiver. Transmitter was mounted on

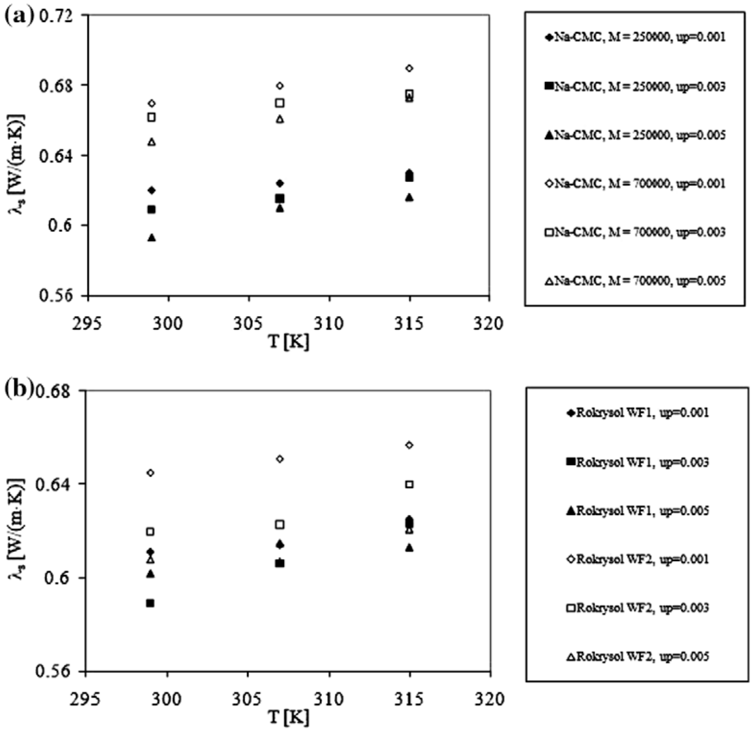

**Fig. 7.** Thermal conductivity for polymer's solutions in static conditions: (a) Na-CMC solutions and (b) polyacrylamide solutions.

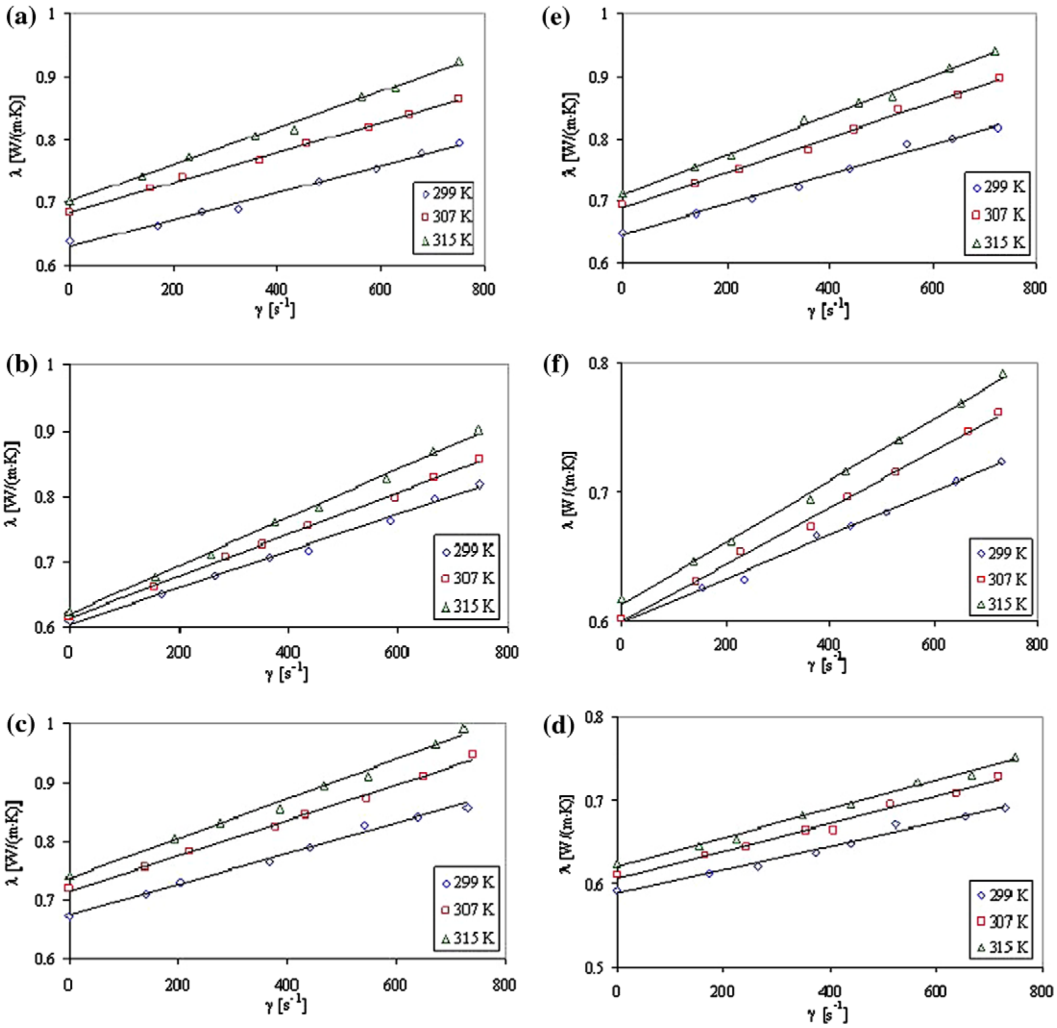

**Fig. 8.** Thermal conductivity $\lambda$ vs. shear rate $\dot{\gamma}$ for mixtures of polymer solutions: (a) mixture I, (b) mixture II, (c) mixture III, (d) mixture IV, (e) mixture V, and (f) mixture VI.

the outer cylinder. Receiver was connected with measurer which indicated temperature on the rotating cylinder. This innovatory element of temperature measurement of the outer cylinder based on wireless communication. Transmitter, changed sensors resistance signal into light signal, which was transmitted from diode. The receive diode, changed light signal into voltage signal, which was registered on measurer. The inner cylinder consisted of the heater, which was designed and realized especially for experiment. The heater consisted of three sections: main and two guards. The two guard-heaters are controlled in such a way to maintain uniform axial temperature in the central, main-heater region. The latter heat flux is virtually in the radial direction only. The guard-heaters will prevent end effects and heat losses. Provided that Fourier‘s law of conduction is applicable, the apparatus is operated at steady state, the shear rate variation across the gap is negligible, and the thermal conductivity of the liquid in the gap is considered to be independent of temperature, the following equation is used to calculate the thermal conductivity:

$$\lambda = \frac{Q \ln\left(\frac{d_o}{d_i}\right)}{2\pi L \Delta T} \tag{11}$$

where $Q$ was calculated from the measurements of current and voltage through the main heater:

$$Q = IV \tag{12}$$

## 3. Experimental results

The liquids tested were water, glycerol (50%) and petrol as Newtonian fluids (Table 3), and aqueous solutions of: carboxymethylcellulose sodium salt (delivered by Aldrich Company, of the molecular masses 250,000 and 700,000 and of the polyelectrolyte

**Table 6**
Averaged critical values of shear rate $\dot{\gamma}_0$.

| Polymer solution | $\dot{\gamma}_0$ (s$^{-1}$) |
|---|---|
| Na-CMC$_{250,000}$, $u_p$ = 0.001 (kg p kg$^{-1}$) | 3 |
| Na-CMC$_{250,000}$, $u_p$ = 0.003 (kg p kg$^{-1}$) | 17 |
| Na-CMC$_{250,000}$, $u_p$ = 0.005 (kg p kg$^{-1}$) | 34 |
| Na-CMC$_{700,000}$, $u_p$ = 0.001 (kg p kg$^{-1}$) | 14 |
| Na-CMC$_{700,000}$, $u_p$ = 0.003 (kg p kg$^{-1}$) | 17 |
| Na-CMC$_{700,000}$, $u_p$ = 0.005 (kg p kg$^{-1}$) | 24 |
| Rokrysol WF1, $u_p$ = 0.001 (kg p kg$^{-1}$) | 3 |
| Rokrysol WF1, $u_p$ = 0.003 (kg p kg$^{-1}$) | 13 |
| Rokrysol WF1, $u_p$ = 0.005 (kg p kg$^{-1}$) | 60 |
| Rokrysol WF2, $u_p$ = 0.001 (kg p kg$^{-1}$) | 30 |
| Rokrysol WF2, $u_p$ = 0.003 (kg p kg$^{-1}$) | 39 |
| Rokrysol WF2, $u_p$ = 0.005 (kg p kg$^{-1}$) | 45 |
| Na-CMC, 0.001 (kg p kg$^{-1}$) + Rokrysol WF1, 0.001 (kg p kg$^{-1}$) | 14 |
| Na-CMC, 0.001 (kg p kg$^{-1}$) + Rokrysol WF2, 0.001 (kg p kg$^{-1}$) | 27 |
| Na-CMC, 0.001 (kg p kg$^{-1}$) + Rokrysol WF1, 0.005 (kg p kg$^{-1}$) | 14 |
| Na-CMC, 0.005 (kg p kg$^{-1}$) + Rokrysol WF1, 0.001 (kg p kg$^{-1}$) | 20 |
| Na-CMC, 0.005 (kg p kg$^{-1}$) + Rokrysol WF2, 0.005 (kg p kg$^{-1}$) | 29 |
| Na-CMC, 0.005 (kg p kg$^{-1}$) + Rokrysol WF1, 0.005 (kg pkg$^{-1}$) | 42 |

concentrations in solutions ranged from 1000 to 5000 w.ppm) and high-molecular polyacrylamide $M \approx (2\text{–}4)\cdot 10^6$ (Rokrysol WF1, delivered by Rokita S.A.), partially hydrolyzed polyacrylamide $M \approx (2\text{–}4)\cdot 10^6$ (Rokrysol WF2, delivered by Rokita S.A.) as well as their mixtures as non-Newtonian fluids (Fig. 6). All polymer solutions used were non-elastic power-law fluids:

$$\tau = K\dot{\gamma}^n \tag{13}$$

Rheological parameters of the polymer's solutions used are shown in Table 4, and characteristics of the polymer's mixtures in aqueous solutions are presented in Table 5. Experiments were performed at temperatures changed in the range from 299 to 315 K. The shear rate range studied was $0 \le \dot{\gamma} \le 750$ [s$^{-1}$]. It was confirmed that thermal conductivity of Newtonian liquids at constant temperature is independent of shear rate. On the ground of experimental studies the relationships $\lambda = f(T)$ for analyzed model Newtonian fluids were found (Table 3).

In Fig. 7 the experimental data of thermal conductivity values obtained in static conditions have been presented. For all non-Newtonian solutions used these values increase with the temperature increase. The maximal values were obtained for aqueous solutions of carboxymethylcellulose sodium salt of the averaged molar mass $M$ = 700,000 (kg kmol$^{-1}$).

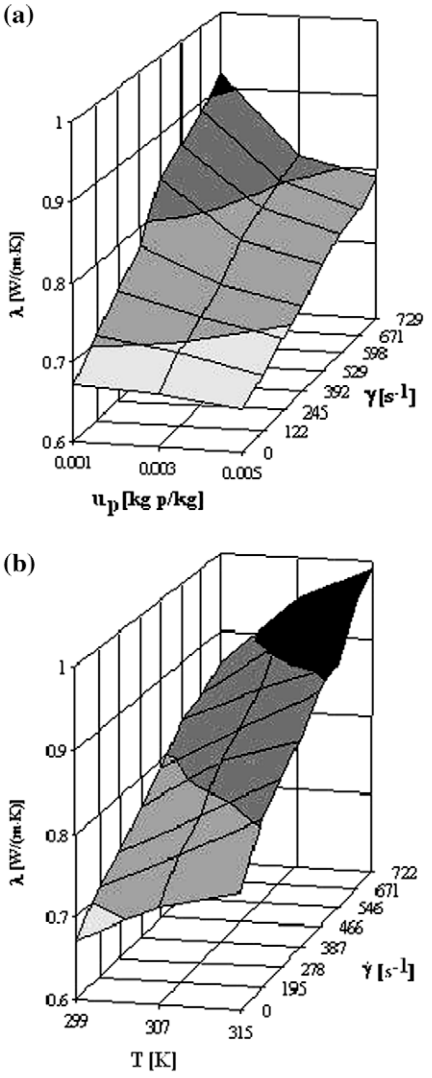

**Fig. 9.** Effect of the shear rate and concentrations in carboxymethylcellulose sodium salt (a) solutions ($M$ = 700,000 (kg kmol$^{-1}$), $T$ = 299 K) and of the shear rate and temperature in mixture III (Na-CMC$_{700,000}$, $u_p$ = 0.001 (kg p kg$^{-1}$) + Rokrysol WF1, $u_p$ = 0.005 (kg p kg$^{-1}$)) aqueous solution.

In Fig. 8 the exemplary experimental relationships of thermal conductivity vs. shear rate for the inelastic non-Newtonian solutions studied are presented. The evident increase of the thermal conductivity compared to water in all polymer solutions used has been observed. The thermal conductivity at a higher temperature at a certain shear rate is greater than at a lower temperature. The temperature effect is significant, and the increase in thermal conductivity at higher temperatures with respect to shear rate is larger than at lower temperatures. The general trends of the curves are similar in all graphs. The solution of the lowest concentration (1000 w.ppm) has a greater change in thermal conductivity with shear rate in comparison with the solution of the highest concentration (5000 w.ppm).

As a result of experimental data elaboration the correlation relationships $\lambda = f(T)$ were determined. The previous observations of Lee and Irvine [15] have been confirmed. Thermal conductivity for polymers and their mixture aqueous solutions increases linearly with increasing shear rate and temperature:

$$\lambda_{T=const} = A'\dot{\gamma} + B' \tag{14}$$

$$\lambda_{\dot{\gamma}=const} = A''_T + B'' \tag{15}$$

The generalized relationship has the form:

$$\lambda = a_o + a_1\dot{\gamma} + a_2T + a_3\dot{\gamma}T \pm 8.6\% \tag{16}$$

The increase of the polymer concentration in a solution caused the decrease of the thermal conductivity $\lambda$. The increase of the molar mass or hydrolyzation degree of a polymer caused the increase in the values of $\lambda$. The maximal values of $\lambda$ = 0.992[W m$^{-1}$ K$^{-1}$) were obtained for the solution of mixture III (Na-CMC$_{700,000}$: $u_p$ = 0.001 (kg p kg$^{-1}$) and Rokrysol WF1: $u_p$ = 0.005(kg p kg$^{-1}$) at temperature of 315 K and shear rate of $\dot{\gamma} = 750(\mathrm{s}^{-1})$.

Additionally the existing of the critical values of shear rate $\dot{\gamma}_0$ below of whose the thermal conductivity for non-Newtonian fluids was independent of shear velocity and equal to the static one, has been observed.

The increase of the shear rate at the values greater than $\dot{\gamma}_0$ caused the effect of the shear rate on thermal conductivity, related to the Shin [14] suggestion. The characteristic values of $\dot{\gamma}_0$ are listed in Table 6. The critical values of shear velocity increase with the polymer concentration in a solution.

Fig. 9a presents the effect of the shear rate and concentrations of carboxymethylcellulose sodium salt solutions, for molecular mass $M$ = 700,000 (kg kmol$^{-1}$) at temperature 299 K. This graph shows that thermal conductivity increased with increasing the shear rate and decreasing the concentration in polymer solutions. The effect of the shear rate and temperature for mixture III aqueous solution is presented in Fig. 9b. The comparison of chosen data for the thermal conductivity obtained in present study with these ones available in literature is presented in Fig. 10. The effect of the shear rate is similar. The divergences can be explained by the fact that various polymers solutions exhibit various behaviour in a flow.

In Fig. 11 the experimental data on non-Newtonian viscosity and thermal conductivity are presented. As the shear rate increases the non-Newtonian viscosity decreases while thermal conductivity increases. All solutions used show shear-thinning properties. It was evidently stated that the thermal conductivity decreased with increasing the viscosity. Next, basing on thermal conductivity and rheological measurements the effect of the non-Newtonian viscosity $\eta$ on thermal conductivity, has been determined. It has been shown that the relation of $\lambda$ vs. $\eta$ depends considerably on molecular mass of the polymer studied and temperature. The effect of the viscosity on thermal conductivity is presented in Fig. 12.

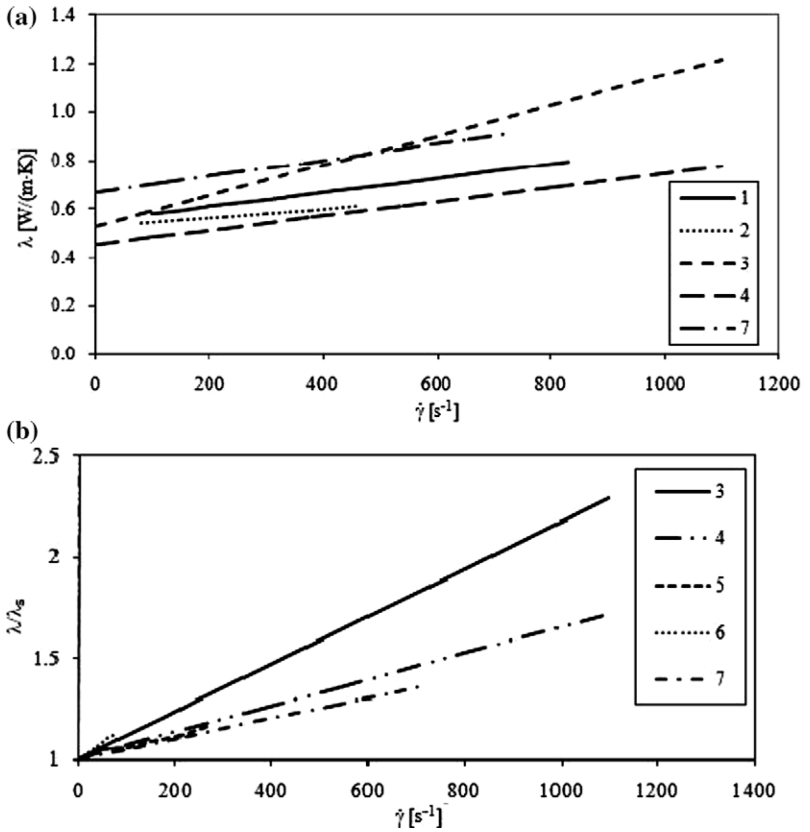

**Fig. 10.** Comparison of the chosen experimental data from present work with relationships suggested in literature: 1 – aqueous solution of Na-CMC 7H4, $u_p = 0.0015$ (kg p kg$^{-1}$), $T = 293$ K (Lee and Irvine [15]); 2 – aqueous solution of Praestol, $u_p = 0.0010$ (kg p kg$^{-1}$), $T = 300$ K (Kostic and Tong [16]); 3 – mango juice concentrated, $T = 305.5$ K (Lin et al. [18]); 4 – orange juice concentrated, $T = 305.5$ K (Lin et al. [18]); 5 – Dow 200, $T = 288.7$ K ([Cocci and Picot [8]); 6 – aqueous solution of Carbopol, $u_p = 0.0010$ (kg p kg$^{-1}$), $T = 298$ K (Loulou et al. [12]); 7 – aqueous solution of Na-CMC, $M = 700{,}000$ (kg kmol$^{-1}$), $u_p = 0.0010$ (kg p kg$^{-1}$), $T = 299$ K, results this study.

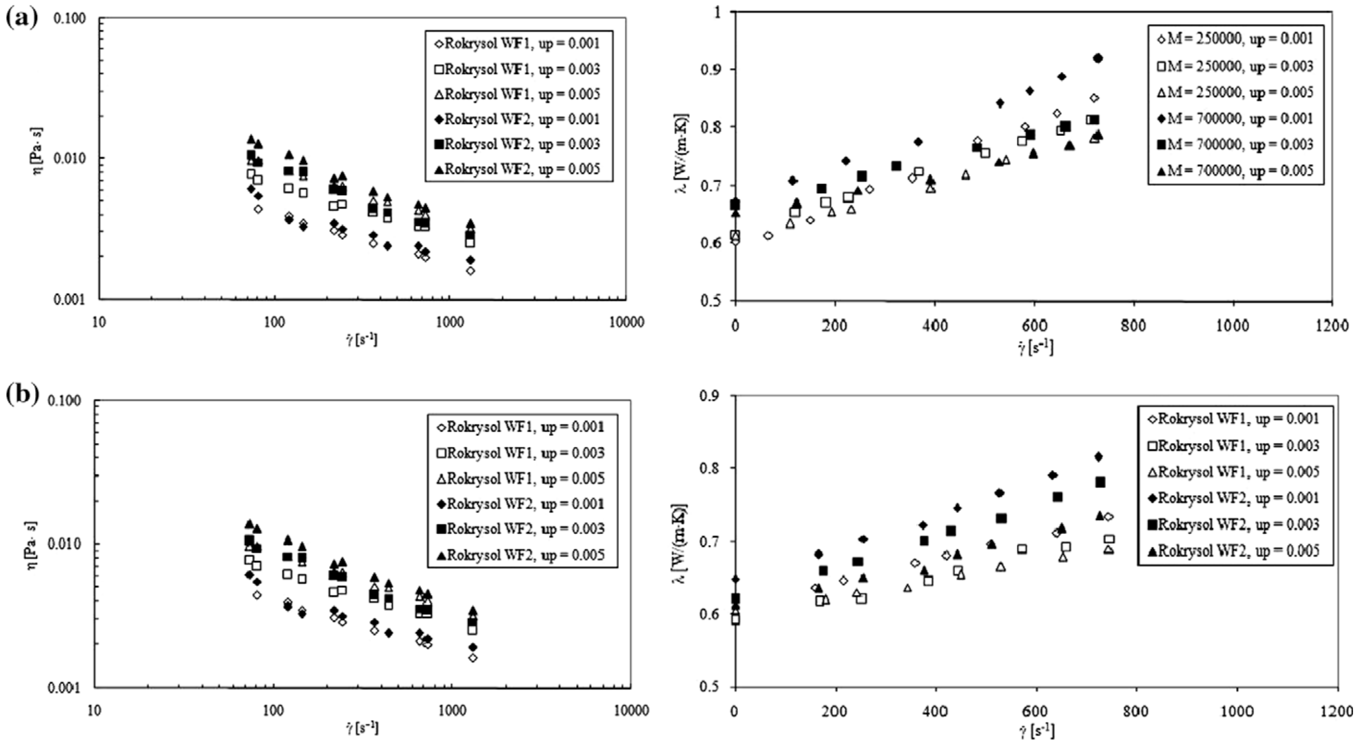

**Fig. 11.** The experimental data on non-Newtonian viscosity and thermal conductivity at 299 K: (a) aqueous solutions of carboxymethylcellulose sodium salt and (b) aqueous solutions of high-molecular polyacrylamide.

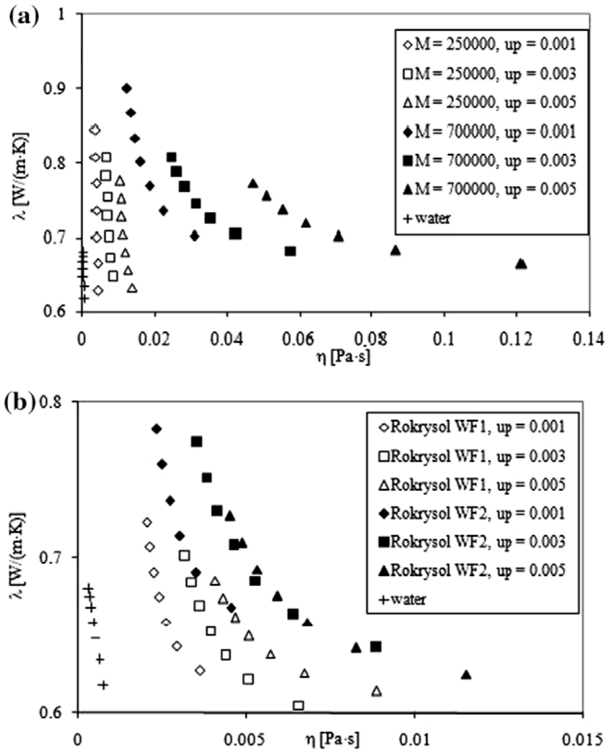

**Fig. 12.** Comparison of the non-Newtonian viscosity effect on thermal conductivity coefficient values at temperature 299 K: (a) aqueous solutions of carboxymethylcellulose sodium salt and (b) aqueous solutions of high-molecular polyacrylamide.

It seems evident that thermal conductivity decreases when the Newtonian or non-Newtonian viscosity increases.

It was affirmed that for all liquids used the relationship of thermal conductivity vs. viscosity can be well approximated by the third degree multinomial function of the form as follows:

$$\lambda = C_0 - C_1\eta + C_2\eta^2 - C_3\eta^3 \tag{17}$$

It should be underlined that for Newtonian fluids the analogous relation exists too. For example:

$$\lambda_{water} = 0.733 - 0.179\eta + 0.0461\eta^2 - 0.000988\eta^3 \tag{18}$$

$$\lambda_{glycerol,50\%} = 0.287 - 0.391\eta + 0.0613\eta^2 - 0.0269\eta^3 \tag{19}$$

In the final step of the experimental data elaboration the special attempt was undertaken to express the relationship between the ratio of non-Newtonian viscosity to thermal conductivity and rheological parameters of polymer solutions used. Basing on the correlations of thermal conductivity vs. shear rate and viscosity vs. shear rate new formula has been proposed for all non-Newtonian fluids:

$$\frac{\eta}{\lambda} = 2.095K^{0.95}\dot{\gamma}^{0.881n-1.021} \pm 15.8\% \tag{20}$$

valid at $n \in (0.51, 0.98)$ and $K \in (0.003, 1.140)$ (Pa s$^n$).

## 4. Conclusions

It has been confirmed that the thermal conductivity for Newtonian fluids is independent of shear rate while it is dependent on temperature. The effect of temperature, shear rate, molecular mass of polymer present in non-Newtonian solution and its concentration on thermal conductivity, has been observed. The increase of the molar mass or hydrolyzation degree of a polymer caused the increase in the values of $\lambda$. It was found that thermal conductivity of polymer's aqueous solutions of shear-thinning properties at constant temperature increased linearly with shear rate. For a given solution studied the thermal conductivity increased with temperature and polymer molecular mass. The increase in concentration of polymer in an aqueous solutions caused the decrease in values of thermal conductivity $\lambda$. It has been stated that the values of thermal conductivity for aqueous solutions of polymers as well as their mixtures increase linearly with the increase of both, shear velocity $\dot{\gamma}$ and temperature. The generalized relationship has the form:

$$\lambda = a_o + a_1\dot{\gamma} + a_2T + a_3\dot{\gamma}T$$

Additionally the existing of the critical values of shear velocity $\gamma_o$ below of whose the values of thermal conductivity for non-Newtonian fluids were independent of shear velocity and equal to the static ones, has been observed. The increase of the shear velocity at the values greater than $\dot{\gamma}_o$ caused the effect of the shear velocity on thermal conductivity coefficient, related to the Shin [14] suggestion.

For all liquid systems tested (Newtonian and non-elastic shear thinning) the decrease of the thermal conductivity with the increase of liquid viscosity was observed. The following correlation between thermal conductivity and Newtonian and non-Newtonian viscosity has been obtained:

$$\lambda = C - C_1\eta + C_2\eta^2 - C_3\eta^3$$

The relationship between the ratio of non-Newtonian viscosity to thermal conductivity and rheological parameters of polymer solutions used is described by the formula:

$$\frac{\eta}{\lambda} = 2.095K^{0.95}\dot{\gamma}^{0.881n-1.021}$$

valid at $n \in (0.51, 0.98)$ and $K \in (0.003, 1.140)$ (Pa s$^n$).

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